

SET 2: Applied Mathematics

Tutorial 8

1. Find the domain of the following functions:

- (a) $f(x) = x^2 - 3x - 2$ [Ans. \mathbb{R}]
(b) $f(x) = \frac{1}{x+2}$ [Ans. $\mathbb{R} \setminus \{-2\}$ or $\mathbb{R} - \{-2\}$ or $(-\infty, -2) \cup (-2, \infty)$]
(c) $f(x) = \frac{2x-3}{x^2-3x+2}$ [Ans. $\mathbb{R} \setminus \{1, 2\}$ or $\mathbb{R} - \{1, 2\}$]
(d) $f(x) = \frac{x^2+3x+5}{x^2-5x}$ [Ans. $\mathbb{R} \setminus \{0, 5\}$ or $\mathbb{R} - \{0, 5\}$]
(e) $f(x) = \sqrt{x-2}$ [Ans. $[2, \infty)$]
(f) $f(x) = \frac{1}{\sqrt{1-x}}$ [Ans. $(-\infty, 1)$]
(g) $f(x) = |x+5|$ [Ans. \mathbb{R}]

2. Find the domain and range of the following functions:

- (a) $f(x) = \frac{1}{\sqrt{x-5}}$ [Ans. Domain: $(5, \infty)$, Range: $(0, \infty)$]
(b) $f(x) = |x-1|$ [Ans. Domain: \mathbb{R} , Range: $[0, \infty)$]
(c) $f(x) = \sqrt{9-x^2}$ [Ans. Domain: $[-3, 3]$, Range: $[0, 3]$]
(d) $f(x) = \sqrt{x^2-16}$ [Ans. Domain: $(-\infty, -4] \cup [4, \infty)$, Range: $[0, \infty)$]

3. Determine whether each of the functions below is even, odd or neither:

- (a) $f(x) = x^6 + x^3 - 2x^2 - 1$ [Ans. Neither even nor odd]
(b) $f(x) = \frac{x^2}{1-x^4}$ [Ans. Even]
(c) $f(x) = \frac{x}{1-x^2}$ [Ans. Odd]

4. If $f(x) = 3x^2 - 4x - 5$, find:

- (a) $f(4)$ [Ans. 27]
(b) $f(-3)$ [Ans. 34]
(c) $f(a)$ [Ans. $3a^2 - 4a - 5$]
(d) $f(2-a)$ [Ans. $3a^2 - 8a - 1$]

5. If $f(x) = \frac{15}{x-3}$, $g(x) = 16 + 3x - x^2$ and $h(x) = \sqrt{25 - x^2}$, find:

(a) $f(0)$ [Ans. -5]

(b) $g(5)$ [Ans. 6]

(c) $h(-4)$ [Ans. 3]

(d) $f(0) + g(4) - h(-3)$ [Ans. 3]

6. Draw the graph the following functions by using the points in the tables below:

(a) $y = f(x) = 2x - 3$

x	-1	0	1	2	3	4
$f(x)$						

(b) $y = f(x) = x^2 - 2x + 3$

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
$f(x)$									

(c) $y = f(x) = -2x^2 + 16x - 31$

x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$									

(d) $y = f(x) = 3^x$

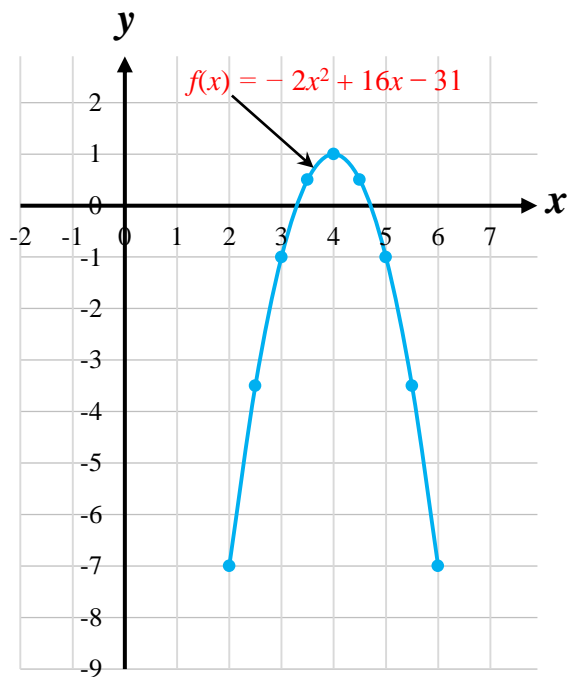
x	-2	-1.5	-1	-0.5	0	1	2
$f(x)$							

(e) $y = f(x) = \log_3 x$

x	0.05	0.1	0.5	1	2	3	4	5	6
$f(x)$									

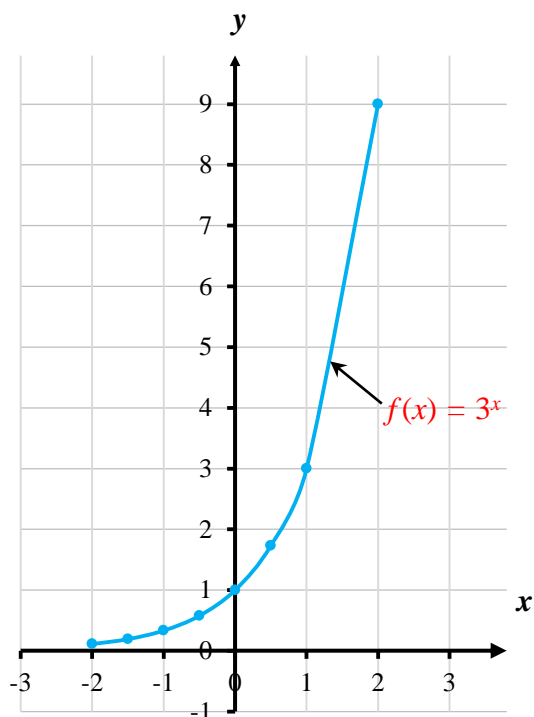
Ans. to problem 6.c

x	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	-7	-3.5	-1	0.5	1	0.5	-1	-3.5	-7



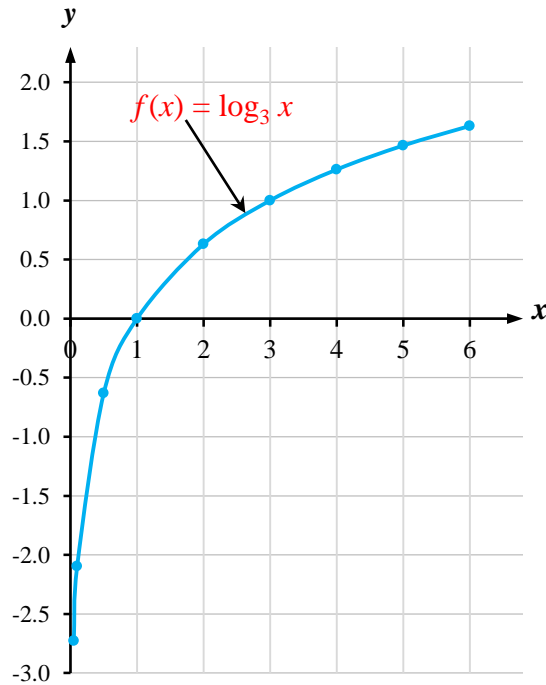
Ans. to problem 6.d

x	-2	-1.5	-1	-0.5	0	0.5	1	2
$f(x)$	0.11	0.19	0.33	0.58	1	1.73	3	9



Ans. to problem 6.e

x	0.05	0.1	0.5	1	2	3	4	5	6
$f(x)$	-2.73	-2.10	-0.63	0	0.63	1	1.26	1.46	1.63



7. For the function of problem 6.b, find:

- (a) the zeros graphically. **[Ans.** No zeros]
- (b) the minimum value graphically. **[Ans.** 2 at $x = 1$]
- (c) the zeros algebraically. **[Ans.** No zeros]
- (d) the minimum value algebraically. **[Ans.** 2 at $x = 1$]

8. For the function of problem 6.c, find:

- (a) the zeros graphically. **[Ans.** $x = 3.3$ and $x = 4.7$]
- (b) the maximum value graphically. **[Ans.** 1 at $x = 4$]
- (c) the zeros algebraically. **[Ans.** $x = 3.3$ and $x = 4.7$]
- (d) the maximum value algebraically. **[Ans.** 1 at $x = 4$]

9. A soft-drink manufacturer has daily production costs of $C = 70,000 - 120x + 0.055x^2$, where C is the total cost (in dollars) and x is the number of units produced. Estimate numerically the number of units that should be produced each day to yield a minimum cost. **[Ans.** $x = 1090$]

10. A field is to be marked off in the shape of a rectangle, with one side formed by a straight river and as shown in the figure below. If 100 m is available for fencing, find the dimensions of the rectangle of maximum possible area. **[Ans.** 25 m by 50 m]

