SET 2

Chapter 7

Simple and Compound Interest

الفائـدة البسيـطة و الفائـدة المركبـة

7.1 Introduction مقدمـة

- Interest rate calculations arise in a variety of business applications, and affect all of us in our personal and professional lives.
- People earn interest on sums they have invested in savings accounts.
- Many home owners pay interest on money they have borrowed for mortgages, personal loans.

7.2 Simple Interest الفائـدة البسيطة

- An amount of money which is invested, or borrowed, is called the principal.
- The amount of interest depends upon the principal, the period of time over which the interest can accumulate, and the interest rate.
- The interest rate is usually expressed as a percentage per period of time, for example 6% per annum.

The simplest form interest can take is called simple interest and is given by the formula:

$$I = P.i\ .n$$ (Simple interest formula)

Where:

I = the interest earned or paid
P = the principal
i = the interest rate per time period, and
n = the number of time periods over which the interest accumulates

Example 1. If an amount of £4000 is invested in a savings account at an interest rate of 8% per year, calculate the simple interest paid over a three year period.

Solution:

P = £4000, $i = 8\% = \frac{8}{100} = 0.08$, and $n = 3$. So,

$I = Pin$

$I = (4000)(0.08)(3)$

I = £960

This is equivalent to earning interest of £960 ÷ 3 = £320 in each of the three years.

7.3 Compound Interest الفائـدة المركبـة

- In the previous example we can imagine earning interest of £320 each year, and withdrawing it immediately it is paid.
- However, in practice the interest earned at the end of each year can be left in the savings account so that it too earns interest.
- This leads to the concept of compound interest.
- In compound interest calculations interest earned, or due, for each period, is added to the principal.
- If a principal P is invested at a rate i per time period, it accrues to an amount S after n time periods given by:

$$S = P\,(1 + i)^n$$ (Compound interest formula)

Example 2. Calculate the amount to which £4000 grows at an interest rate of 8% per annum, for three years, if all the interest earned is reinvested.

Solution:

The interest rate is 8% = $\frac{8}{100}$ or 0.08

$$S = P\,(1+i)^n$$
$$S = 4000(1 + 0.08)^3$$
$$S = £5038.85$$

Example 3. A person invests \$1500 in a two-year bond paying 4.5% interest per year. Money is left in the account for the whole of the two-year period. Assuming compound interest, what amount will be in the account at the end of the two-year period?

Solution:

Amount invested = P = \$1,500
Interest rate = i = 0.045 p.a. (4.5%)
Number of time periods = n = 2

$$S = P(1+i)^n$$
$$S = 1500(1+0.045)^2$$
$$S = 1500(1.045)^2$$
$$S = \$1638.04$$

Amount paid out after two years is \$1,638.04

Example 4. Assuming compound interest, calculate the interest earned by investing:
(a) OMR 500 for 3 years at 4% per annum
(b) OMR 400 for 2 years at 0.5% per month
(c) OMR 300 for 1 year at 2% per half-year.

Solution:

(a) P = OMR 500, i = 0.04 and n = 3

$$S = P(1+i)^n$$
$$S = 500(1+0.04)^3$$
$$S = 500(1.04)^3$$
$$S = \text{OMR } 562.43$$

Interest earned = 562.43 – 500 = OMR 62.43

(b) P = OMR 400, $i = 0.005$ and $n = 24$ (2 years = 24 months)

$S = P(1+i)^n$

$S = 400(1+0.005)^{24}$

$S = 400(1.005)^{24}$

S = OMR 450.86

Interest earned = 450.86 – 400 = OMR 50.86

(c) P = OMR 300, $i = 0.02$ and $n = 2$ (1 year = 2 half-years)

$S = P(1+i)^n$

$S = 300(1+0.02)^2$

$S = 300(1.02)^2$

S = OMR 312.12

Interest earned = 312.12 – 300 = OMR 12.12

Example 5. Rashid has OMR 10,000 to invest for one year and is considering three different accounts:

(a) A one year bond offering 4% per annum
(b) An account offering 0.35% per month
(c) An account offering 2.1% per half-year.

He does not need the interest until the end of the year. Assuming compound interest, into which account should he invest his money to maximize the interest?

Solution:

Using $S = P(1+i)^n$ in each case gives:

(a) P = OMR 10,000, $i = 0.04$ and $n = 1$

Then, $S = 10000(1+0.04)^1$
$S = 10000(1.04)$
S = OMR 10400

Interest earned = 10400 – 10000 = OMR 400

(b) P = OMR 10,000, $i = 0.0035$ and $n = 12$

$S = 1000(1+0.0035)^{12}$

$S = 10000(1.0035)^{12}$

S = OMR 10428.2

Interest earned = 10428.2 – 10000 = OMR 428.2 ✓

(c) P = OMR 10,000, $i = 0.021$ and $n = 2$

$S = 10000(1+0.021)^2$

$S = 10000(1.021)^2$

$S = \text{OMR } 10424.4$

Interest earned = 10424.4 – 10000 = OMR 424.4

Therefore, the account that offers 0.35% per month is the best for Rashid to invest his money.

Example 6. How much does Ali need to invest today at 2.9% compounded semi-annually to save OMR 14,000 after 3 years?

Solution:

Amount invested today = P = ?
Interest rate = i = 0.029 per half-year (2.9%)
Number of time periods = n = 6
Amount in the account after 3 years = S = OMR 14,000

$$14000 = P(1+0.029)^6$$
$$14000 = P(1.029)^6$$
$$14000 = P(1.18711)$$

$$P = \frac{14000}{1.18711}$$
$$P = \text{OMR } 11{,}793.3$$

7.4 Continuous Compounding الفائدة المركبة المستمرة

- Interest earned on an investment, or due on a loan, is usually compounded.
- Compound interest was previously described when the compounding period was annual, semi-annual, monthly...etc.
- On occasions, interest is compounded continuously which has the effect of increasing the amount of interest.
- When interest is compounded continuously, the accrued amount at any time t is $S(t)$ and is given by:

$$S(t) = P_0 e^{it}$$ (Continuous compounding interest formula)

Where P_0 = the principal invested right at the start
e = the exponential constant
i = interest rate

- When using this formula, the units of time must be consistent throughout. So for example, if i is the annual interest rate, t must be measured in years.

Example 7. A principal of £1000 is invested at a constant annual rate of 8%. Interest earned is compounded continuously. Find the accrued amount after 25 years.

Solution:

$P_0 = 1000$, $i = 0.08$ and $t = 25$ we have:

$$S(25) = 1000 \times e^{(0.08 \times 25)} = 1000 \times e^2 = \text{£}7{,}389.06$$

Example 8. Suppose that $2000 is invested at interest rate *i*, compounded continuously, and grows to $2504.65 in 5 years.

(a) What is the interest rate?
(b) Find the exponential growth (continuous compounding) function $S(t)$.
(c) What will the balance be after 10 years?
(d) After how long will the $2000 be doubled?

Solution:

(a) At $t = 0$, $S(0) = P_0 = \$2000$.
So, the exponential growth function is of the form:

$$S(t) = 2000e^{it}$$

We know that $S(5) = \$2504.65$. We substitute and solve for i:

$$2504.65 = 2000e^{i(5)}$$
$$2504.65 = 2000e^{5i}$$
$$\frac{2504.65}{2000} = e^{5i}$$
$$\ln\frac{2504.65}{2000} = \ln e^{5i}$$
$$\ln\frac{2504.65}{2000} = 5i$$
$$i = 0.045$$

The interest rate is 0.045 or 4.5%.

(b) By substituting 0.045 for i in the function $S(t) = 2000e^{it}$:

$$S(t) = 2000e^{0.045t}$$

(c) The balance after 10 years is:

$$S(10) = 2000e^{0.045(10)} = 2000e^{0.45} = \$3136.62$$

(d) To find the doubling time T:

$S(T) = 2 \times P_0 = 2 \times \$2000 = \$4000$

Solve for T:

$$4000 = 2000e^{0.045T}$$
$$\frac{4000}{2000} = e^{0.045T}$$
$$2 = e^{0.045T}$$
$$\ln 2 = \ln e^{0.045T}$$
$$\ln 2 = 0.045T$$
$$T = \frac{\ln 2}{0.045}$$
$$T = 15.4 \text{ years}$$

Thus, the original investment of \$2000 will double in about 15.4 years.