

SET 2

Chapter 6

Linear Equations and Inequalities in Two Variables

المعادلات و المتباينات الخطية بمتغيرين

6.1 Linear Equations in Two Variables

المعادلات الخطية بمتغيرين (المعادلات الأنوية بمتغيرين)

- Two linear equations in two variables are **solved simultaneously**.
- Therefore, they are called two **simultaneous** (أنوية) linear equations in two variables, or a **system** of two linear equations in two variables.
- Simultaneous equations are **solved**:
 - (i) By substitution **بالتعويض**
 - (ii) By elimination **بالحذف**
 - (iii) Graphically **بالرسم البياني**
 - (iv) By matrices and determinants **بالمصفوفات و المحددات**

6.2 Solution of Two Simultaneous Linear Equations by Substitution

حل معادلتين أنيتين خطيتين بمتغيرين بالتعويض

Example 1. Solve the following simultaneous equations by substitution and check your results:

$$\begin{aligned}x + 6y &= 8 \\ 2x - 5y &= -1\end{aligned}$$

Solution:

$$x + 6y = 8 \quad (1)$$

$$2x - 5y = -1 \quad (2)$$

From equation (1):

$$x + 6y = 8$$

$$x = 8 - 6y$$

Substitute $(8 - 6y)$ for x in equation (2):

$$2x - 5y = -1 \quad (2)$$

$$2(8 - 6y) - 5y = -1$$

$$16 - 12y - 5y = -1$$

$$-17y = -1 - 16$$

$$-17y = -17$$

$$y = 1$$

By substituting 1 for y in equation (1) or equation (2), the value of x may be found. Use equation (1) to find x :

$$x + 6y = 8 \quad (1)$$

$$x + 6(1) = 8$$

$$x + 6 = 8$$

$$x = 8 - 6$$

$$x = 2$$

Checking the answer by substituting 2 for x and 1 for y in equation (2) gives:

$$\text{LHS} = 2(2) - 5(1) = 4 - 5 = -1 = \text{RHS}$$

Thus, the solution is $x = 2$ and $y = 1$.

Example 2. Use the substitution method to solve the following simultaneous equations and verify the results:

$$-15r + 7t = -1$$

$$2r + t = 4$$

Solution:

$$-15r + 7t = -1 \quad (1)$$

$$2r + t = 4 \quad (2)$$

From equation (2):

$$2r + t = 4 \quad (2)$$

$$t = 4 - 2r$$

Substituting $(4 - 2r)$ for t in equation (1) gives:

$$-15r + 7t = -1 \quad (1)$$

$$-15r + 7(4 - 2r) = -1$$

$$-15r + 28 - 14r = -1$$

$$-29r = -1 - 28$$

$$-29r = -29$$

$$r = 1$$

Substitute 1 for r in equation (2) to find t :

$$2r + t = 4 \quad (2)$$

$$2(1) + t = 4$$

$$2 + t = 4$$

$$t = 4 - 2$$

$$t = 2$$

Check the solution by substituting $r = 1$ and $t = 2$ in equation (1):

$$-15r + 7t = -1 \quad (1)$$

$$-15(1) + 7(2) \stackrel{?}{=} -1$$

$$-15 + 14 \stackrel{?}{=} -1$$

$$-1 = -1 \quad \text{Yes} \Rightarrow \text{The solution is } r = 1 \text{ and } t = 2.$$

Example 3. Solve the following simultaneous equations by substitution and check the results:

$$2x - 3y = 17$$

$$4x + 5y = 1$$

Solution:

$$2x - 3y = 17 \quad (1)$$

$$4x + 5y = 1 \quad (2)$$

From equation (1):

$$2x - 3y = 17$$

$$2x = 3y + 17$$

$$x = \frac{3y + 17}{2}$$

Substitute $\frac{3y + 17}{2}$ for x in equation (2):

$$4x + 5y = 1 \quad (2)$$

$$4\left(\frac{3y + 17}{2}\right) + 5y = 1$$

Multiply both sides by 2:

$$4(3y + 17) + 10y = 2$$

$$12y + 68 + 10y = 2$$

$$22y = 2 - 68$$

$$22y = -66$$

$$y = \frac{-66}{22}$$

$$y = -3$$

Substitute $y = -3$ in any of the two equations to find x . Use equation (1) to find x :

$$2x - 3y = 17 \quad (1)$$

$$2x - 3(-3) = 17$$

$$2x + 9 = 17$$

$$2x = 17 - 9$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

Verify the results by substituting 4 for x and -3 for y in equation (2):

$$\text{LHS} = 4(4) + 5(-3) = 16 - 15 = 1 = \text{RHS}$$

Hence, the solution is $x = 4$ and $y = -3$.

6.3 Solution of Two Simultaneous Linear Equations by Elimination

حل معادلتين أنيتين خطيتين بمتغيرين بالحذف

Example 4. Solve the following simultaneous equations by elimination and verify the results:

$$2x + 3y = 8$$

$$x + 2y = 3$$

Solution:

$$2x + 3y = 8 \quad (1)$$

$$x + 2y = 3 \quad (2)$$

To eliminate x , multiply equation (2) by 2 first:

$$x + 2y = 3 \quad (2)$$

$$2(x + 2y) = 2(3)$$

$$2x + 4y = 6 \quad (3)$$

Then subtract equation (3) from equation (1):

$$2x + 3y = 8 \quad (1)$$

$$2x + 4y = 6 \quad (3)$$

$$\begin{array}{r} \text{Subtract} \\ \hline 0 - y = 2 \end{array}$$

Thus $y = -2$

To find x , substitute $y = -2$ in equation (1):

$$2x + 3y = 8 \quad (1)$$

$$2x + 3(-2) = 8$$

$$2x - 6 = 8$$

$$2x = 8 + 6$$

$$2x = 14$$

$$x = \frac{14}{2}$$

$$x = 7$$

Check the solution by substituting 7 and -2 for

x and y respectively in equation (2):

$$x + 2y = 3 \quad (2)$$

$$(7) + 2(-2) \stackrel{?}{=} 3$$

$$7 - 4 \stackrel{?}{=} 3$$

$$3 = 3 \quad \text{Yes}$$

Therefore, the solution is $x = 7$ and $y = -2$.

Example 5. Solve the following simultaneous equations by elimination and verify the results:

$$3a - 4b = -22$$

$$7a + 3b = -2$$

Solution:

$$3a - 4b = -22 \quad (1)$$

$$7a + 3b = -2 \quad (2)$$

Multiply equation (1) by 3:

$$3a - 4b = -22 \quad (1)$$

$$3(3a - 4b) = 3(-22)$$

$$9a - 12b = -66 \quad (3)$$

Then multiply equation (2) by 4:

$$7a + 3b = -2 \quad (2)$$

$$4(7a + 3b) = 4(-2)$$

$$28a + 12b = -8 \quad (4)$$

To eliminate b , add equations (3) and (4):

$$9a - 12b = -66 \quad (3)$$

$$28a + 12b = -8 \quad (4)$$

$$\begin{array}{r} \hline \text{Add} \quad 37a + 0 = -74 \end{array}$$

$$\text{Thus } a = \frac{-74}{37} = -2$$

Substituting -2 for a in equation (1) gives:

$$3a - 4b = -22 \quad (1)$$

$$3(-2) - 4b = -22$$

$$-6 - 4b = -22$$

$$-4b = -22 + 6$$

$$-4b = -16$$

$$b = \frac{-16}{-4} = 4$$

Check the results by substituting $a = -2$ and $b = 4$ in equation (2):

$$7a + 3b = -2 \quad (2)$$

$$7(-2) + 3(4) \stackrel{?}{=} -2$$

$$-14 + 12 \stackrel{?}{=} -2$$

$$-2 = -2 \quad \text{Yes} \Rightarrow \text{The solution of the system is } a = -2 \text{ and } b = 4.$$

6.4 Solution of Two Simultaneous Linear Equations Graphically

حل معادلتين أنيتين خطيتين بمتغيرين بالرسم البياني

To solve a system of two linear equations in two variables graphically:

- **Draw** the graphs of the two equations on the same graph.
- The graphs are **two straight lines** that **intersect** at a specific point.
- The **point of intersection** of the two lines represents **the solution** of the system.

Example 6. Solve the following simultaneous equations graphically and verify the results:

$$2x - y = 5$$

$$2x + y = 3$$

Solution:

Draw equation (1) by finding the **x-intercept** and the **y-intercept**:

x-intercept

$$\begin{aligned} 2x - y &= 5 & (1) \\ 2x + (0) &= 5 \\ x &= 2.5 \end{aligned}$$

y-intercept

$$\begin{aligned} 2x - y &= 5 & (1) \\ 2(0) - y &= 5 \\ y &= -5 \end{aligned}$$

Use **(2.5, 0)** and **(0, -5)** to draw equation (1).

Similarly, use the **x-intercept** and the **y-intercept** to graph equation (2):

x-intercept

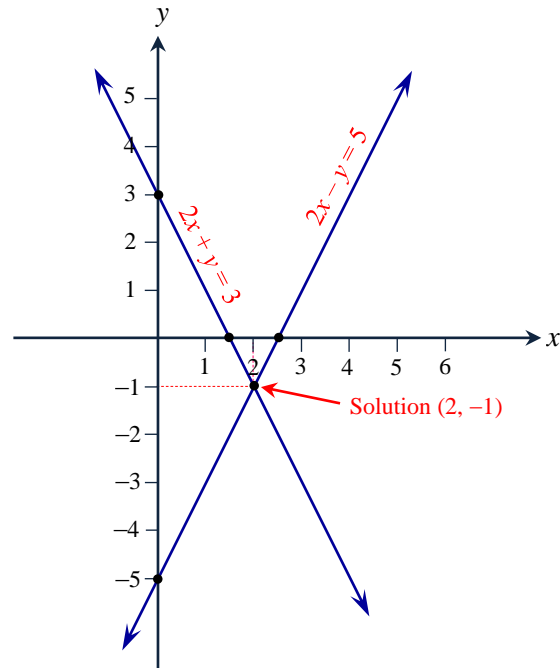
$$\begin{aligned} 2x + y &= 3 & (2) \\ 2x + (0) &= 3 \\ x &= 1.5 \end{aligned}$$

y-intercept

$$\begin{aligned} 2x + y &= 3 & (2) \\ 2(0) + y &= 3 \\ y &= 3 \end{aligned}$$

Use **(1.5, 0)** and **(0, 3)** to draw equation (2).

The graph of the two equations is shown in the following figure.



From the graph, $x = 2$ and $y = -1$

Verify the results:

$$2x - y = 5 \quad (1)$$

$$\text{LHS} = 2(2) - (-1) = 4 + 1 = 5 = \text{RHS}$$

$$2x + y = 3 \quad (2)$$

$$\text{LHS} = 2(2) + (-1) = 4 - 1 = 3 = \text{RHS}$$

Hence, the solution of the system is $x = 2$ and $y = -1$.

6.5 Linear Inequalities in Two Variables

المتباينات الخطية بمتغيرين

- The solution of linear inequalities in two variables x and y is all points (x, y) that satisfy the inequality.

Example 7. Find the solution for $4x - 3y \leq 12$ graphically.

Solution:

1- Draw the graph of $4x - 3y = 12$:

x-intercept

$$4x - 3y = 12$$

$$4x - 3(0) = 12$$

$$x = 3$$

y-intercept

$$4x - 3y = 12$$

$$4(0) - 3y = 12$$

$$y = -4$$

Thus, use $(3, 0)$ and $(0, -4)$ to draw the equation $4x - 3y = 12$ as a **solid line**.

2- Choose a **random point** other than the boundary line points to test.

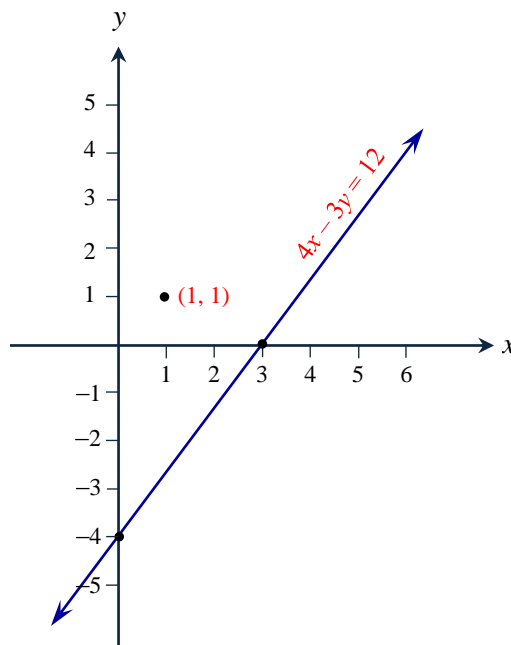
Test $(1, 1)$:

$$4x - 3y \leq 12$$

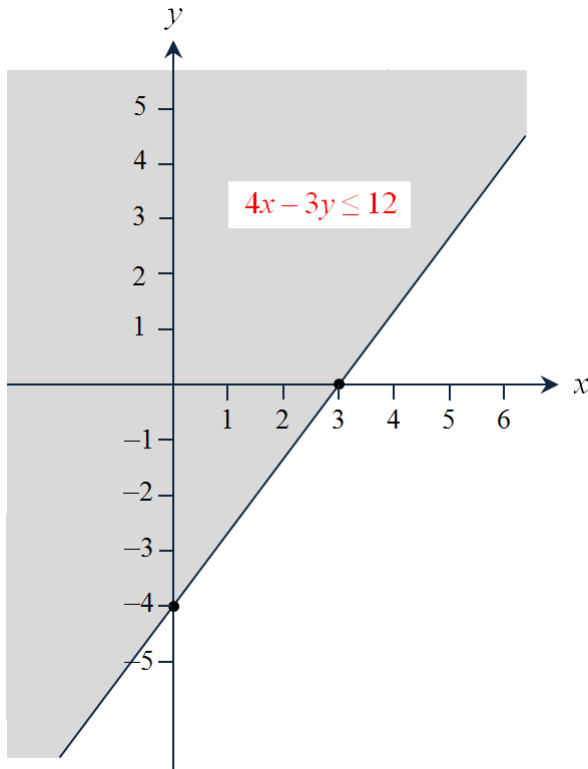
$$4(1) - 3(1) \stackrel{?}{\leq} 12$$

$$4 - 3 \stackrel{?}{\leq} 12$$

$7 \leq 12$ Yes \Rightarrow The region above the boundary line is the solution region.



3- Shade the **solution region** as in the figure below.



Example 8. Graph the solution to the following system of inequalities:

$$x + 2y \leq 12$$

$$3x - y \geq 6$$

Solution:

To graph the solution to a system of inequalities:

- 1- Graph each inequality on the same graph.
- 2- See where the shading of the inequalities overlaps. The overlapping region is the solution region to the system of inequalities.

For $x + 2y \leq 12$, use $(12, 0)$ and $(0, 6)$ to draw the boundary line, and then **test point** $(0, 0)$:

$$x + 2y \leq 12$$

$$(0) + 2(0) \stackrel{?}{\leq} 12$$

$0 \leq 12$ Yes \Rightarrow The region **below** the boundary line is the **solution region**.

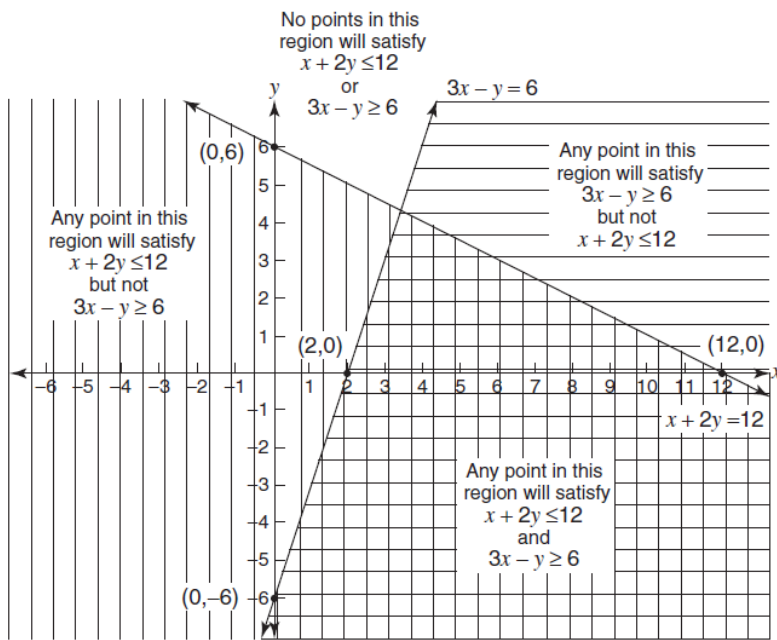
For $3x - y \geq 6$, use $(2, 0)$ and $(0, -6)$ to draw the boundary line, and then **test point** $(0, 0)$:

$$3x - y \geq 6$$

$$3(0) - (0) \stackrel{?}{\geq} 6$$

$0 \geq 6$ No \Rightarrow The region **below** the boundary line is the **solution region**.

The solution to the system is the region that contains the shading of the two inequalities and as shown in the figures below.



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