SET 1

Chapter 7

Quadratic Equations

المعـادلات التربـيعيـة

7.1 Introduction مقدمة

- A quadratic equation is one in which the highest power of the unknown quantity is 2.
- For example, $x^2 - 3x + 1 = 0$ is a quadratic equation.
- The general form of quadratic equations is:

$$ax^2 + bx + c = 0$$

- There are five methods for solving quadratic equations:
  (i) by square root. بأخذ الجذر التربيعي
  (ii) by factorisation (where possible). بالتحليل
  (iii) by using the quadratic formula. بالقانون العام
  (iv) by completing the square. بإكمال المربع
  (v) graphically. بالرسم البياني

7.2 Solution by the Quadratic Formula الحل بإستخدام القانون العام

- The Quadratic formula is written as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1. Solve $x^2 + 2x - 8 = 0$ by using the quadratic formula.

Solution

For the equation $x^2 + 2x - 8 = 0$:

$a = 1$, $b = 2$ and $c = -8$.

Substituting these values into the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+32}}{2} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$

$$x = \frac{-2+6}{2} \quad \text{or} \quad \frac{-2-6}{2}$$

So, $\boldsymbol{x = \frac{4}{2} = 2}$ or $\boldsymbol{x = \frac{-8}{2} = -4}$

Example 2. Solve $4x^2 + 7x + 2 = 0$ by using the quadratic formula.

Solution

For the equation $4x^2 + 7x + 2 = 0$:

$a = 4$, $b = 7$ and $c = 2$.

Substituting these values into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(4)(2)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 - 32}}{8} = \frac{-7 \pm \sqrt{17}}{8}$$

$$x = \frac{-7 + \sqrt{17}}{8} \quad \text{or} \quad \frac{-7 - \sqrt{17}}{8}$$

Therefore, $\boldsymbol{x = \dfrac{-7 + \sqrt{17}}{8}}$ or $\boldsymbol{x = \dfrac{-7 - \sqrt{17}}{8}}$

7.3 Real Life Examples مسائل من الواقع

Example 3. The length of a rectangular piece of land is 4 m longer than its width, and its area is 45 m^2. Find the dimensions of this land.

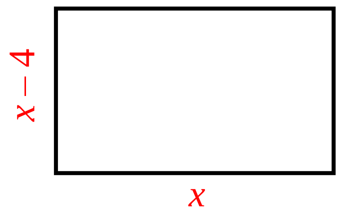
Solution

Let the length = x $\Rightarrow$ the width = $x - 4$

The area = length × width

The area = $x(x - 4) = 45$

$x^2 - 4x - 45 = 0$



$a = 1$, $b = -4$ and $c = -45$.

Substituting these values into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-45)}}{2(1)}$$

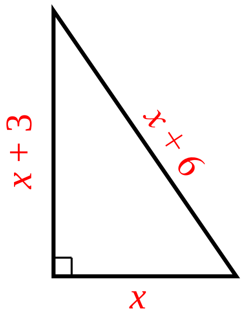
$$x = \frac{4 \pm \sqrt{16 + 180}}{2} = \frac{4 \pm \sqrt{196}}{2} = \frac{4 \pm 14}{2}$$

$$x = \frac{4 + 14}{2} = 9 \quad \text{or} \quad x = \frac{4 - 14}{2} = -5 \text{ (ignore)}$$

Therefore, length = 9m and the width = 9 – 4 = 5m

Example 4 The hypotenuse of a right-angled triangle is 6 cm longer than the shorter leg, and the longer leg is 3 cm more than the shorter leg. Find the length of each side of this triangle.

Solution



Let the length of the shorter leg = x

Then, the length of the longer leg = $x + 3$,

and the length of the hypotenuse = $x + 6$

Applying Pythagoras theorem gives:

$$(x + 6)^2 = x^2 + (x + 3)^2$$

$$x^2 + 12x + 36 = x^2 + x^2 + 6x + 9$$

$$x^2 - 6x - 27 = 0$$

$a = 1$, $b = -6$ and $c = -27$.

Substituting these values into the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 108}}{2} = \frac{6 \pm \sqrt{144}}{2} = \frac{6 \pm 12}{2}$$

$$x = \frac{6 + 12}{2} = 9 \quad \text{or} \quad x = \frac{6 - 12}{2} = -3 \text{ (ignore)}$$

Therefore, the shorter leg = 9 cm ,

the longer leg = 9 + 3 = 12 cm ,

and the hypotenuse = 9 + 6 = 15 cm